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MATHEMATICS AND THE *SECUNDUM IMAGINATIONEM* PROCEDURE IN RICHARD KILVINGTON[*]

The aim of this paper is to present Richard Kilvington's methodology, based on mathematics and the *secundum imaginationem* procedure, in order to reveal the novelty of his theories, as well as to answer the question about originality of his works. While answering this question, one can give to two different responses. First, that Kilvington was an original thinker who created new ideas and concepts and invented new methodology and procedures in logic, natural philosophy, ethics and theology, which were later used and developed by medieval and pre-modern thinkers. Therefore, Kilvington can justly be called a creative philosopher and theologian. Second, that Kilvington was a representative of the fourteenth-century English Oxford Calculators,[1] and that his theories simply reflect a way of thinking common to this group of scholars. In fact, both explanations are correct. On the one hand, in Kilvington's works one finds a great number of lengthy, elaborated, fully developed discussions and original solutions to philosophical and theological problems. On the other, in Kilvington's works, one finds a palette of typical questions and problems discussed at that time and solved in logico-mathematical manner, which was par for the course for the Oxford Calculators' School. Therefore, it might seem that we could,

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[1] On Oxford Calculators see E. Jung[-Palczewska], *Między filozofią przyrody i nowożytnym przyrodoznawstwem. Ryszard Kilvington i fizyka matematyczna w średniowieczu* [*Between Philosophy of Nature and Modern Science. Richard Kilvington and Mathematical Physics in the Middle Ages*], Łódź: Wydawnictwo Uniwersytetu Łódzkiego, 2002, pp. 330. In this book a reader will find information on primary and secondary literature concerning the School of Oxford Calculators.

following John Murdoch, classify Kilvington's way of thinking as analytical language, and the problems he solves as metalinguistic ones.[2] If we, however, accept that interpretation we might overlook that which constitutes the core of Kilvington's thought. We might also interpret some of his hypotheses from the point of view of modern science, drawing a bridge between the Middle Ages and post-Newtonian science. This, however, seems to be problematic, primarily due to the entirely different methodology used by modern as opposed to and medieval scientists.

Richard Kilvington was born at the very beginning of the fourteenth century and died, most likely, as a victim of the second outbreak of the Black Death in England in 1361. My detailed study of Kilvington's works and historical documents from the era made it possible to establish a chronology of his works and his biography. During the years 1316–26, Kilvington was a student and regent-master at the Faculty of Arts at Oxford University, lecturing on logic and on Aristotle's *On the Generation and Corruption, Physics* and *Ethics.* In the next years, he was a student at the Faculty of Theology, lecturing on the *Sentences* in 1333–34. The next year he spent in Durham as a member of the circle of Richard of Bury. In 1334, he became a Bachelor of Theology, and, before 1337, a Doctor of Theology. The following year, he was sent with a diplomatic mission to the Continent and left Oxford for good. Kilvington's academic career was only a relatively short stint early in his life; it began when he was approximately eighteen years old and ended when he was about thirty-three. It served as a launching pad for a successful diplomatic and ecclesiastical career, which, as it seems, was his true objective. In 1350, Kilvington became Archdeacon of London; in 1354, he was appointed as a Dean of Saint Paul's Cathedral in London.[3]

We do not know of any philosophical or theological works written by Kilvington after his transition from the university to a public career. It seems that his diplomatic and ecclesiastical career did not stimulate further scholarship, nor did his membership in Richard of Bury's household.[4] The only works we have at our disposal are the results of Kilvington's lecturing at Oxford. Even though

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[2] See J.E. Murdoch, *The Analytical Character of Late Medieval Learning: Natural Philosophy without Nature*, [in:] *Approaches to Nature in the Middle Ages*, l. D. Roberts (red.), Binghamton, N.Y., 1982, pp. 171–213; Idem, *The Involvement of Logic in Late Medieval Natural Philosophy*, [in:] *Studies in Medieval Natural Philosophy*, S. Caroti (ed.), Firenze, 1989, pp. 3–23.

[3] For detailed information see E. Jung[-Palczewska], *Works by Richard Kilvington*, „Archives d'Histoire Doctrinale et Littéraire du Moyen Age," 67 (2000), pp. 184–225.

[4] William Courtenay maintains, on the contrary, that: "Those who did not make the transition from university to public career in church, state, or religious order produced any writings after leaving the schools (...) By contrast, the burdens of high office did not prevent and may even have stimulated further scholarship..." (see W.J. Courtenay, *Schools and Scholars in Fourteenth-Century England*, Princeton, 1987, p. 146).

his works are commentaries to Aristotle and to Peter Lombard, none of them follows the order of books as presented by the Philosopher or Lombard themselves. In accordance with fourteenth-century practice at Oxford, the number of topics Kilvington discusses, is reduced to those which are most essential and suitable for the then-new mathematical methodology subjects. All of his commentaries are planned and constructed as sets of fully developed questions (no more than ten). The reduction of the range of topics is counterbalanced by an increased intensity of analysis in the question chosen for treatment. Some of his questions are spread over fifteen folios, which, in modern editions, yields about 120 pages. All his philosophical works (*Sophismata*, *Quaestiones super De generatione et corruptione*, *Quaestiones de motu*, *Quaestiones super Ethicam*) were composed before 1326; his questions on the *Sentences* appeared before 1334.

The form of a question is almost always the same: first, Kilvington formulates the main problem in the form of a question starting "*Utrum…*," usually followed by *probo quod non*. Then, he presents a contrary opinion, which, except for the questions on the *Sentences*, is based on the authority of Aristotle and Averroes (in his *Sentences* he recalls Saint Augustine). *Contra* is followed by a *solutio quaestionis*. The last part of a question contains *responsiones* to principal arguments. Frequently, principal arguments are fully developed debates, and sometimes *dubia* are inhered in the discussion. The material presented resembles vivid discussions from the classroom, so it gives the contemporary reader the impression that these are *reportata* of debates held by students. The discussions also closely resemble the scheme of Kilvington's first logical work — *Sophismata*.[5]

In his questions on Aristotle's *Libri naturales*, Kilvington considers the issues connected with constant changes, such as local motion, qualitative and quantitative changes by means of the following terms: *incipit, desinit, tempus, instans, gradus, spatium, intensior, velocius, remissior, pars proportionalis, divisum,* and others — just as he does in his *Sophismata*. Many issues that derive from his logic and philosophy of nature are discussed also in his ethics and theology.[6] Kilvington's pragmatic attitude towards ethical and theological problems is evident at the outset. Most of the questions debate the issues of temporal changes and

[5] We already have at our disposal Kilvington's *Sophismata* (see B.E. Kretzmann, N. Kretzmann (eds.), *The Sophismata of Richard Kilvington*, Oxford, 1991), questions on the *Ethics* (see Richard Kilvington's *Quaestiones super libros Ethicorum*: A Critical Edition with Introduction by M. Michałowska, Leiden: Brill, 2016); translation into English of his *Sophismata* (see *The Sophismata of Richard Kilvington: Introduction, Translation and Commentary*, New York: Cambridge University Press, 1991); Polish of his *Quaestiones de motu* (see E. Jung, *Arystoteles na nowo odczytany. Ryszarda Kilvingtona "Kwestie o ruchu"*, Łódź: Wydawnictwo Uniwersytetu Łódzkiego, 2014).

[6] For his ethics see M. Michałowska, *Woluntarystyczny dynamizm. Koncepcja woli w "Kwestiach do Etyki" Ryszarda Kilvingtona*, Kraków: Księgarnia Akademicka, 2016.

processes, such as decreases or increases in love (*dilectio*), enjoyment (*fruitio*), and pleasure (*delectatio*), the infinite capacity of the soul for grace, the augmentation of grace, and temporal or eternal reward or punishment. For Kilvington, logic and mathematics seem to be the most suitable methods to be used to measure such seemingly abstract entities as, for instance, a venial sin or right judgment.[7]

The use of mathematics to solve logical, physical, ethical and theological problems is a special mark of Kilvington's methodology. The application of mathematics to change allows him to determine — that is, to "measure" — the phenomena in question. His "measure mania," to use John Murdoch's expression,[8] is meaningfully related to his special interest in the description of different types of changes. The measuring of qualitative changes or changing things is introduced first in Kilvington's *Sophismata*, which consists of 48 conceptual problems. Sophisms 1 through 44, devoted to issues of natural philosophy (*sophismata physicalia*), appear to have physical subjects. The type of measuring presented in *Sophismata* is focused on establishing limits to the beginning and ending of continuous processes such as Socrates's whitening or increasing speed in motion. This type of measure, by first and last instance, does not appear straightforwardly mathematical. Yet it deals with mathematical considerations, since it prescribes a measure for natural processes.

A similar way of measuring, by limits, also appears in Kilvington's questions on motion. His solution to the limit-decision problem reveals, at the outset, the originality of his philosophy of nature. This is seen best in the long and detailed discussions presented in his first question on motion, where he establishes the criteria for intrinsic (*maximum quod sic, minimum quod sic*) and extrinsic (*maximum quod non, minimum quod non*) limits of passive and active potencies; that is, a maximum and a minimum they can or cannot accomplish. Kilvington rejects the most popular (Aristotelian) opinion, stating that the range of an active potency can be settled by a *maximum quod sic*, that is, by its intrinsic limit, and proves that a capacity of an active potency has to be determined by the extrinsic limit of a *minimum passum* that it cannot accomplish. On the other hand, limits for passive potency should be determined with regard to circumstances; that is, sometimes it should be a *minimum passum* that can be accomplished, while in other circumstances it should be a maximum which cannot be accomplished

[7] For theology see, E. Jung – M. Michałowska, *Scotistic and Ockhamist Contribution to Kilvington's Ethical and Theological Views*, [in:] *1308 Ein Topographie historischaer Gleichzeitigkeit*, „Miscellanea Mediaevalia," Bd. 35 (2010), pp. 104–125.

[8] See J.E. Murdoch, *The Analytical Character of Late Medieval Learning: Natural Philosophy without Nature*, [in:] *Approaches to Nature in the Middle Ages*, L.D. Roberts (ed.), Binghamton, N.Y., 1982, pp. 171–213.

by an active potency. Kilvington explains this in the following way: sometimes, sight is considered to be strong because it can see tiny pieces, but sometimes (if we, for example, are standing too close to a cathedral), we cannot see the whole building. Thus, in the first case, the limit for the passive potency will be a *minimum quod sic*, while in the second case it is a *maximum quod non*. It is worth noting that Kilvington points out that the boundaries for active and passive potencies must be perceptible.[9] This suggests that we can determine the range of active and passive potencies, as well as their limits, only by observing the process of changing. Kilvington often uses this type of measuring — by assigning limits — in his ethical and theological questions, too. For example, he employs it in discussions dealing with changes of moral and theological virtues.

Most of the arguments invented by Kilvington were used later by Wiliam Heytesbury, who presents "the most astute analyses of the limit-decision problem," as Curtis Wilson says.[10] Heytesbury, however, transfers the discussion to a metalinguistic level while maintaining that, "it is the kinds of *terms* that occur in the very formulation of the problem at hand that will enable one to decide whether a *maximum quod sic* or a *minimum quod non* is appropriate."[11] It seems that also Roger Rosethus, who broaches the issue in the initial question of his commentary on the *Sentences*, was familiar with Kilvington's solution of the limit-decision problem.[12]

Additionally, in all of Kilvington's works, one finds a second type of "measuring": by latitude of forms. This type of measuring serves as a methodological tool in solving problems connected with the increasing and decreasing intensity of accidental forms in such processes as heating or becoming white. Kilvington devotes his second question on motion specifically to this problem.[13] In his *Sentences* he also frequently uses this type of measuring in order to establish the latitude of moral and theological qualities such as love, hate, grace, sin, and so on, or to clarify the nature of communication between God and man in general, and God and the blessed in particular. These debates place Kilvington in the mainstream of fourteenth-century theological discussions.

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[9] See Ryszard Kilvington, *Kwestie o ruchu*, [in:] E. Jung, *Arystoteles na nowo odczytany*, q. 1, pp. 109–171, esp. pp. 124–133, 156–166.

[10] C. Wilson, *William Heytesbury. Medieval Logic and the Rise of Mathematical Physics,* Madison: University of Wisconsin Press, 1960, p. 87.

[11] Quote from J. Longway, *Wiliam Heytesbury on Maxima and Minima. Chapter 5 of 'Rules for solving sophismata' with an anonymous fourteenth-century discussion*, Dordrecht, 1984, p. 158.

[12] Roger Rosethus, *Quaestiones super libros Sententiarum*, q. 1, *Utrum aliquis in causa possit obligari ex praecepto ad aliquid quod est contra conscientiam suam*, Ms. Oxford, Oriel College 15, f. 227v.

[13] See Ryszard Kilvington, *Kwestie o ruchu*, [in:] E. Jung, *Arystoteles na nowo odczytany*, q. 2, pp. 173–248.

The third type of measuring, strictly mathematical, employs a new calculus of proportions (compounding ratios). This is present in all of Kilvington works, but one finds the broad explanation of such calculus in his first question from *Questions on motion*, *Utrum in omni motu potentia motoris excedit potentiam rei motae*.[14] By means of this new calculus of ratios, it is possible to determine variations of speed with respect to force and resistance. In order to produce a mathematically coherent theory, Kilvington, like Bradwardine after him,[15] argues that, according to Euclid's definition from the fifth Book of the *Elements*, a proper double proportion is a multiplication of a proportion by itself; since, in Averroes's opinion, the ratio of speeds in motion follows the ratios of the power of the mover to the power of the thing moved, the variations of speeds must be tied to variations in the proportion of forces and resistances. Consequently, the proper rule for the calculus of speed in motion is, in modern terms, the exponential-type function,[16] which was — in Kilvington's opinion — what Aristotle and Averroes had in mind when they formulated the rules for motion.[17]

The fourth and the last type of measuring is also Kilvington's original invention. Using modern terminology, we can say that he describes the rule allowing one to "measure" infinite sets containing infinite subsets. In accordance with Aristotle, each continuum — such as space or time — can be infinitely divided, and as such it is potentially infinite. Aristotle states that infinities must be equal. Kilvington nicely proves, however, that there are unequal infinities which differ *secundum quid*. Since each *continuum* can be infinitely divided, it consists of infinite parts that are also made up of infinite parts, and so on *ad infinitum*. Thus, in modern terms, each *continuum* is an infinite set containing infinite subsets that are unequal with regard to the unequal cardinality of the sets involved. Consequently, the mathematical rules and axioms for finite quantities can be applied, without any change, to infinite quantities. As a result, we come to the "calculus" of infinities by means of which we can compare infinities and determine which of them are equal, lesser or greater than others. This brilliant idea reveals that

[14] See Ryszard Kilvington, *Kwestie o ruchu*, [in:] E. Jung, *Arystoteles na nowo odczytany*, q. 1, pp. 134–154, 167–171.

[15] On the interdependency between Richard Kilvington and Thomas Bradwardine, E. Jung, *Arystoteles na nowo odczytany*, pp. 39–46.

[16] On the new calculus of ratio see for example: J. Murdoch, *The Medieval Language of Proportions: Elements of the Interaction with Greek Foundations and the Developement of New Mathematical techniques*, [in:] *Scientific Change. Historical studies in the intellectual, social and technical conditions for scientific discovery and technical invention, from antiquity to the present*, A.C. Crombie (ed.), London, 1963, pp. 237–271; E.D. Sylla, *Compounding ratios. Bradwardine, Oresme, and the first edition of Newton's* Principia, [in:] *Transformation and Tradition in the Sciences.* (Essays in honor of I. Bernard Cohen), Cambridge, 1984, pp. 11–43.

[17] Ryszard Kilvington, *Kwestie o ruchu*, [in:] E. Jung, *Arystoteles na nowo odczytany*, q. 1, p. 168.

the fundamental axiom, "the part is lesser than the whole," is no longer valid, since infinities do not have to be equal.[18] This is the basic argument in George Cantor's set theory, devised in the nineteenth century.

Kilvington finds use for all of the above-mentioned types of "measuring" while describing the processes that occur in the real world and in the imaginable one. In his first question on motion, he explains his scientific method as follows:

> It happens that the thoughts, which seem to be probable, are sometimes false and sometimes true. Therefore, having carefully debated the opinions of others, I weigh and uphold those that can be confirmed by the most evident reasons.[19]

This declaration clearly shows that Kilvington is aware that statements in natural philosophy are more or less probable and not necessarily demonstrative, since the principles are not evident. Only logic is a demonstrative science, and its statements are valid irrespective of their reference to the outside reality or imagination. Hence, Kilvington never uses the expression *secundum imaginationem* in his logical treatise *Sophismata*. Imaginable cases, however, are found in all his works describing the natural and human world.

Imaginable cases can be grouped in two. The first group consists of those imaginable examples that can be observed in nature. The best example here comes from his first question on motion, which questions the foundation of the Aristotelian rule of motion grounded on the assumption that the doubling of an acting power doubles the speed of motion. Kilvington's counter-argument is as follows:

> Suppose that someone carrying a bean runs as fast as he can, and then someone else joins him, and he also runs as fast as he can with the same speed, and they both carry the same bean. Thus, the doubled acting power does not cause a double speed of motion,[20] because they cannot run faster.

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[18] See R. Podkoński, *Nieskończoność w ujęciu Ryszarda Kilvingtona*, Łódź: Wydawnictwo Uniwersytetu Łódzkiego, 2016.

[19] Ricardus Kilvington, *Quaestiones de motu*, q. 1, *Utrum in omni motu potentia motoris excedit potentiam rei motae*, Ms. Venice, Biblioteca San Marco, lat VI, 72 (2810), f. 89ra: „Quedam argumenta in ista materia valent ad opposita, que dissolvi potuerunt per iam dicta per illos articulos propositos. Plures sapientes opiniones contrarias et ambiguas habuerunt. Et ubi sapientes discordant, difficile est veritatem investigare. Nam quedam falsa sunt probabiliora, quedam vera; et falsum quandoque propter apparentias veri plures iudicant esse verum. Ideo opiniones aliorum in hac materia diligenter studens dixi recitandas et ponderandas; hinc de rationibus dictarum opinionum velut ponderibus in brachiis equlibre, ille opiniones firmius teneantur que evidentioribus rationibus poterunt confirmari."

[20] Ibidem, f. 84rb: „Item posito quod aliquis homo trahat unam fabam per unam cordam currendo ita velociter sicut potest, tunc si alius homo tante potentie ad currendum sibi iniungatur

This group also includes all of the imaginable cases that are not observable in nature, but which belong to its realm. The best example here is the motion of the Earth. Kilvington argues as follows: The sides (or hemispheres) of the Earth are not equally heavy, since: 1) mountains and valleys are unsymmetrically distributed on its surface, 2) in some parts, the Earth is flat, 3) in some parts, there is water, which makes the Earth a friendlier place for animals, 4) the sun heats and dries some of the Earth's parts more than others. Therefore, the Earth moves, because one of its parts is heavier than the other, it loses its center of gravity, which should be in the same place as the center of the universe. Thus, the Earth continuously moves to reach the universe's center of gravity; it does not naturally stay in its proper place.[21]

Kilvington is aware that this conclusion is opposite to Aristotle's point of view, and he explains as follows:

> It is true that Aristotle states that the Earth should be fixed in the center, yet what he wants to say is that the Earth does not revolve on its axis, and not that it cannot move with rectilinear motion.[22]

Finally, Kilvington concludes that the Earth's motion is so slow that it is not significant.

The second group of examples applies the *secundum imaginationem* procedure to tentative cases only. By means of mathematics, Kilvington, in the realm of

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ad trahendum illam fabam predictam, illi duo homines non trahent velocius quam unus illorum per se. Ergo velocitas motus non sequitur excessum."

[21] Ibidem, f. 86vb–87ra: „Sed dico, quod omnibus aliis paribus concedi potest conclusio et iuxta sententiam Philosophi, quod terra est centrum in continuo motu. Et quando dicitur, quod Philosophus vult in *De motu animalium*, quod omne motum in suo motu necessario indiget fixo, ideo centrum indiget terra quiescente — dico quod intelligit, quod terra quiescit a tali motu circulari quali movetur celum. Non enim volo ponere, quod terra moveatur motu circulari eternaliter circa suum centrum, sicut posuit Plato de terra consimiliter sicut de igne probat (recitatur sua opinio secundo *De celo*, commento 96). Sed dico, quod ceteris paribus terra movetur eternaliter, dicendo aliter, motibus secundum partem graviorem, verum tamen motu valde tardo et insensibili. Et quiescit terra a motu sensibili non obstante, quod insensibiliter moveatur. Et hec opinio satis videtur sequi ex sententia et processu Philosophi secundo *De celo* versus finem illis locis allegatis. Dico etiam ultra, quod si terra naturaliter debet esse sperica est tam vallosa et in quibusdam partibus plana, propter convenientiorem habitudinem animalium, sicut aqua congregata in certis locis, non obstante, quoddam terram undique circumdare, ita quod ista [circumstantia] sunt quodammodo violenta et aliqualiter propter naturam, non tamen violenta propter peius sed propter melius, et talia violenta sunt sive possunt esse eterna. Sed alia violenta corrumpentia vel destruentia, que sunt propter peius et non propter melius, non sunt eterna. Et sic intelligit Aristoteles secundo *De celo* textu commenti 17, 18 et 19 et Commentator sic intelligit illis commentis."; Ryszard Kilvington, *Kwestie o ruchu*, [in:] E. Jung, *Arystoteles na nowo odczytany*, pp. 120–121, 156–158.

[22] Ricardus Kilvington, *Quaestiones de motu*, q. 1, *Utrum in omni motu potentia motoris excedit potentiam rei motae*, Ms. Venice, Biblioteca San Marco, lat VI, 72 (2810), f. 86vb (cf. Supra).

speculation, makes apparent the paradoxes arising from Aristotle's theory of motion. Here the best example is Kilvington's third question on motion: *Utrum aliquod corpus simplex possit moveri aeque velociter in vacuo et in pleno*.[23] The title of the question informs us that the it author is not interested in proving the possible existence of a void, nor even possible motion in a void, but in explaining the motion of a simple body (such as a piece of earth) in a vacuum. Generally speaking, Kilvington is interested in describing the physical process that would occur if a vacuum existed. In the whole question, one finds only tentative cases, which allow him to solve, by means of mathematics, some detailed problems and *dubia*. Like later Oxford Calculators, Kilvington adopts Ockham's position of ontological minimalism and claims that, since a void is not observable in nature, there no evidence that it exists. Nevertheless, since there is no obstacle either from nature or from God for a vacuum to exist, it is only a point of speculation to invent a type of mathematical physics that would justify a possible motion of both: composed and simple bodies in a void.

Further into Kilvington's *secundum imaginationem* procedure, one learns of four levels of consideration of imaginable cases. These levels can be characterized by increasing abstraction and decreasing probability. On the first level, there are the real cases occurring and observable in nature; on the second, cases that are not observable, but can occur in nature (like the Earth's rectilinear motion); on the third, cases that are not observable, but that are theoretically possible (such as infinite speed in an instant); on the fourth level, tentative cases that are only theoretically possible (such as motion in a vacuum). The *secundum imaginationem* procedure allows Kilvington to reveal inconsistencies in Aristotelian philosophy of nature, and so to fabricate counter-arguments in his discussion. To be sure, Kilvington contrasts things that are really distinct with things that are distinct only mentally, i.e., in the imagination. If a hypothetical case does not involve contradiction, there is no reason to reject it from the realm of speculation, and thus the term "imaginable" stands in for the term "reasonable." While Kilvington accepts the Aristotelian view of the world, and the principles laid down in his natural philosophy, he nevertheless introduces mathematics into physics and develops mathematical physics in order to overcome the paradoxes emerging from Aristotle's laws. Yet even if Kilvington is perfectly aware of the inconsistencies in Aristotle's theories, he does not offer any new theory to replace them.

In my opinion, Kilvington's ideas gave later Oxford Calculators, as well as some of the Continental philosophers, an impulse to develop a proper

[23] Ricardus Kilvington, q. 3, *Utrum aliquod corpus simplex possit moveri aeque velociter in vacuo et in pleno*, Ms. Venice, Biblioteca San Marco, lat VI, 72 (2810), ff. 101ra–107vb; Ryszard Kilvington, *Kwestie o ruchu* [in:] E. Jung, *Arystoteles na nowo odczytany*, pp. 249–288.

secundum imaginationem procedure and theoretical physics. Later on, William Heytesbury included, in his *Regulae solvendi sophismata*, the entire range of imaginable cases. Just like Kilvington, he claims that the only requirement for an imaginable case is that it should not involve a formal, logical contradiction. Heytesbury develops Kilvington's concept, and he draws a distinction between *realiter* and *naturaliter* and a *physice loquendo* procedure, in which we follow observation and the principles laid down in Aristotle's natural philosophy, and *logice* and *sophistice loquendo*, under which we are free to introduce whatever distinction and imaginable cases may be convenient. In the opinion of Heytesbury, it is a matter of indifference whether such cases are physically possible.[24]

Roger Swineshead and John Dumbleton — the next generation of Oxford Calculators — further developed the procedure *secundum imaginationem*. Dumbleton tries to discover correct mathematical physics and to develop a mathematical science of motion. He is convinced that the proper method is to abstract quantities mentally and to deal with them mathematically. Thus we can, for example, imagine space outside the world — says Dumbleton — if we separate quantity from matter body, even though they are never separated in reality. The result of this procedure will be a three-dimensional space. According to Dumbleton, things do not follow imagination, and thus such considerations belong only to the realm of imagination. Nevertheless, Dumbleton's idea of empty, absolute space outside the real world was later developed by Isaac Newton as a fundamental principle of his physics.[25]

Roger Swineshead, on the other hand, sees a great role for human reason and creativity in devising a mathematical physics. He develops a *secundum imaginationem* procedure and frequently uses the term "imagination" as a synonym for "reason." He opposes these terms to "reality". Swineshead presents a conceptualist attitude toward the relation between mathematics and physics, which can be observed in his use of the term "imagination" with reference to modes of beings and not to categories that refer to the beings themselves; that is, with reference to hypothetical cases that do not exist in the real world, but are posited as parts of disputations.[26]

In Kilvington, the *secundum imaginationem* procedure differs considerably from the reasoning *de potentia Dei absoluta*. Frequently, in the secondary

[24] See C. Wilson, *William Heytesbury. Medieval Logic and the Rise of Mathematical Physics*, pp. 24–25.

[25] See E.D. Sylla, *Imaginary Space: John Dumbleton and Isaac Newton*, „Miscellanea Mediaevalia," Bd. 25 (1998), pp. 206–225.

[26] See E.D. Sylla, *Mathematical physics and imagination in the work of the Oxford Calculators: Roger Swineshead's On Natural Motion*, [in:] *Mathematics and its implications to science and natural philosophy in the Middle Ages*, E. Grant, J. Murdoch (eds), Cambridge Univ. Press, 1987, pp. 85–96.

literature, these two methods are considered to be the same. Kilvington rightly observes that the *secundum imaginationem* method finds its use in the sciences whose subjects belong to the realm of nature, like physics or ethics. On the other hand, the *de potentia Dei absoluta* method finds its proper use in theology, dealing with God, His actions, His essence, and the relation between God-as-Creator and the world. Kilvington is sure that God, by His absolute power, can overcome the obstacles stemming from nature, and can bring about, for example, infinite, temporal motion, or reduce the World to the dimensions of a single bean. The only requirement is not to violate the rule of non-contradiction. Kilvington, like other Oxford Calculators, refrained from including God in his speculations in natural science and focused on nature — the proper subject of physics. Nevertheless, it was obvious to him that the laws of nature reflect God's ordained power.[27] There is no need to summon God while searching for the laws of nature or deliberating mental experiments.

Kilvington's *secundum imaginationem* procedure is frequently accompanied by a *ceteris paribus* method, which resembles a Galilean procedure of idealization in scientific inquiry. McMullin describes the aim of this kind of idealization as "grasping the real-world base from which the idealization takes its origin"[28] by making the problem simpler and therefore more tractable. This procedure resulted in proper solutions of scientific problems. While he does use this procedure, Kilvington has no problem with mathematical justifications of, for instance, a motion in a void.

To sum up, Kilvington's *secundum imaginationem* procedure involves both logic and mathematics. The former is the method that makes it possible to determine the coherency of a given case — used when one is trying to resolve doubts about to whether a case is possible or impossible. The latter, mathematics, was used to create methods for measuring qualitative and quantitative changes (both finite and infinite). It provided a handy tool for constructing theories that could successfully be verified. It seems that Kilvington, like his great predecessor Robert Grosseteste, was convinced that mathematics is a proper method for doing science. He was also certain that mathematical science has practical implications. I am thus quite certain that Kilvington's *secundum imaginationem* procedure does not belong to the world of metalinguistic analysis. Instead, Kilvington follows Ockham's logico-critical approach. He concentrated

[27] See for example E. Jung – M. Michałowska, *Scotistic and Ockhamist Contribution to Kilvington's Ethical and Theological Views*, [in:] *1308 Ein Topographie historischer Gleichzeitigkeit*, „Miscellanea Mediaevalia," Bd. 35 (2010), pp. 104–125; E. Jung, *Świat możliwy versus świat realny w średniowiecznych koncepcjach, czyli o Boskiej mocy absoluta i ordinata*, "Filo-Sofija," 30 (2015/3), pp. 76–78.

[28] E. McMullin, *Galilean Idealization*, "Studies in History and Philosophy of Science," 16 (1985), pp. 247–273.

on empirical, logical and mathematical evidence to ensure a firmer foundation for philosophy. The subject, however, needs further research.

MATHEMATICS AND THE *SECUNDUM IMAGINATIONEM* PROCEDURE IN RICHARD KILVINGTON

Summary

This paper is focused on the methodology of Richard Kilvington, one of the founders of the Oxford Calculators School in the fourteenth century. This methodology is based on mathematics and the *secundum imaginationem* procedure. Mathematics, used by Kilvington to solve all kind of problems belonging to logic, physics, ethics and theology, is a special trait of Kilvington's methodology. He is especially interested in describing different types of changes that can occur in the real world and in imaginary ones. Kilvington finds uses for all types of "measuring" of different changes, which was a popular intellectual endeavor in his time. Kilvington uses the *secundum imaginationem* procedure to prove that mathematics is a proper method in doing philosophy of science. He is certain that mathematical science has practical implications. Kilvington accepts Ockham's logico-critical approach, which focused on empirical, logical and mathematical evidence in order to ensure a firmer foundation for philosophy.

Keywords: Richard Kilvington, Medieval Science, *secundum imaginationem*, methodology

Słowa klucze: Ryszard Kilvington, nauka średniowieczna, *secundum imaginationem*, metodologia